

Instability of the Kolmogorov flow in a soap film

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We examine the instability of a soap film flow driven by a time-independent force that is spatially periodic in the direction perpendicular to the forcing (Kolmogorov flow). Linear stability analysis of an idealized model of this flow predicts a critical Reynolds number $R_c \approx \sqrt{2}$. In our soap film experiment, we find a critical value $R_c \approx 70$. This discrepancy can be ascribed to frictional effects from viscous coupling of gas to the film, which is neglected in the idealized model. The kinematic viscosity of the surrounding gas and the thickness of gas layers on each side of the soap film are varied in the experiments to better understand these frictional effects. Our observations indicate that flow in the soap film cannot be decoupled from flow in the surrounding gas. [S1063-651X(99)04307-X]

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I. INTRODUCTION

The flow in a flat, freely suspended soap film lies in the plane of the film. Experiments have recently used soap films to study two-dimensional flows, including grid-generated turbulence and vortex shedding [1–6]. However, three-dimensional effects due to the viscous coupling of a soap film to the surrounding gas remain poorly understood. We present an experiment that clearly demonstrates the importance of this issue: the primary instability of a spatially periodic flow in a horizontal soap film.

A soap film consists of two monolayers of surfactant molecules bounding a 0.1–10 μm thick layer of water/surfactant solution. A film may be driven into motion by, for example, gravity or viscous coupling to a moving gas. For any forcing method, the viscous coupling to the surrounding gas can be non-negligible. The thinness of the film causes the viscous flow of the interstitial layer relative to the surfactant monolayers to be slow compared to the flows of the film as a whole. The difference in time scales for the flow of the film and the flow of the interstitial fluid means that we can treat the interstitial fluid and its surrounding monolayers as a single two-dimensional fluid. Mysels *et al.* [7] and Couder *et al.* [8] discuss the properties of soap films and the conditions under which they may be considered two-dimensional fluids.

While the soap film itself behaves as a two-dimensional fluid, the viscous coupling between the film and the surrounding gas is a three-dimensional effect. Our results indicate that flow in a soap film cannot be decoupled from flow in the surrounding gas.

Consider a two-dimensional fluid with an imposed spatially periodic body forcing, $\mathbf{F}_{\text{body}} = F_0 \sin(ky) \hat{\mathbf{x}}$, where x and y lie in the plane of the film. Kolmogorov suggested this forcing geometry as a toy problem to study the transition to turbulence in an ideal two-dimensional fluid [9]. The resulting base flow $\mathbf{U}(y) = U \sin(ky) \hat{\mathbf{x}}$ is called Kolmogorov flow and is predicted to become unstable at a critical Reynolds

number $R_c = U/k\nu = \sqrt{2}$, where ν is the kinematic viscosity [10,11]. The instability of the Kolmogorov flow has been studied numerically [12] and experimentally [13]. The experiment consisted of a thin layer of electrolytic fluid on a glass plate in a spatially varying magnetic field. A current, passed through the fluid, drove the Kolmogorov flow. Bondarenko *et al.* found that the system became unstable at $R_c \approx 10^3$. This discrepancy between experiment and linear stability analysis was attributed to the friction between the glass plate and the fluid. By taking this three-dimensional friction into consideration, Thess [14] later conducted a numerical linear stability analysis that yielded $R_c \approx 10^3$, in agreement with the experiment. Systems which can be modeled with a two-dimensional equation of motion but in which three-dimensional effects remain important have been called quasi-two-dimensional [15]. This magnetohydrodynamic system could not be described by a strictly two-dimensional model, but was successfully described by a quasi-two-dimensional model.

We study the instability of the Kolmogorov flow in a soap film. We apply a quasi-two-dimensional model to describe the viscous coupling between the film and the surrounding gas. Figure 1(a) shows the Kolmogorov flow below onset of instability in the soap film system. At a critical forcing amplitude, the flow becomes unstable to a pattern of steady-state vortices with three or four vortices in each row [see Fig. 1(b)].

II. EXPERIMENTAL APPARATUS

We implement the Kolmogorov flow by mechanically forcing a soap film as shown in Fig. 2. A horizontal film stretched across a metal frame is suspended over a set of belts with alternate belts moving in opposite directions. The belts pull the gas above them, and due to a viscous coupling between the film and the gas, the film is driven into motion as well.

The belts supply a forcing that is periodic, but not exactly sinusoidal. Figure 3 shows hot film anemometry measurements of the horizontal velocity profile in the gas above the belts in the absence of a soap film. Farther above the belts, the velocity profile more closely approximates a sinusoid.

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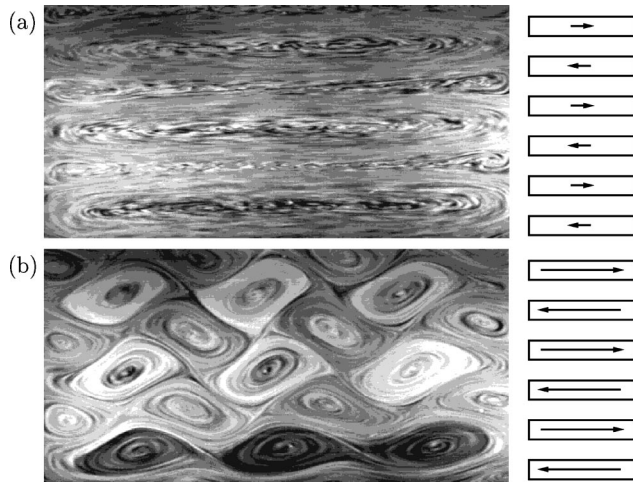


FIG. 1. Soap film image (a) below and (b) well above the primary instability; the boxes on the right indicate the locations and directions of the moving belts under the film (cf. Fig. 2). The intensity variations in the images correspond to thickness variations in the film. These images have been averaged over one second to enhance the underlying structure of the fluid flow for the purposes of illustration; the vortex turnover time is approximately four seconds for these images. Each image ($12.5 \text{ cm} \times 7.5 \text{ cm}$) has been cropped to reduce the importance of end effects during analysis.

The belts are spaced such that a zero-velocity line in the Kolmogorov flow coincides with the side walls of the frame from which the soap film is suspended.

The soap film is suspended horizontally at a distance h_b above the belts; h_b is adjustable in the range $0.2\text{--}1.0 \text{ cm}$. The belts move around pulleys with a horizontal axis and are

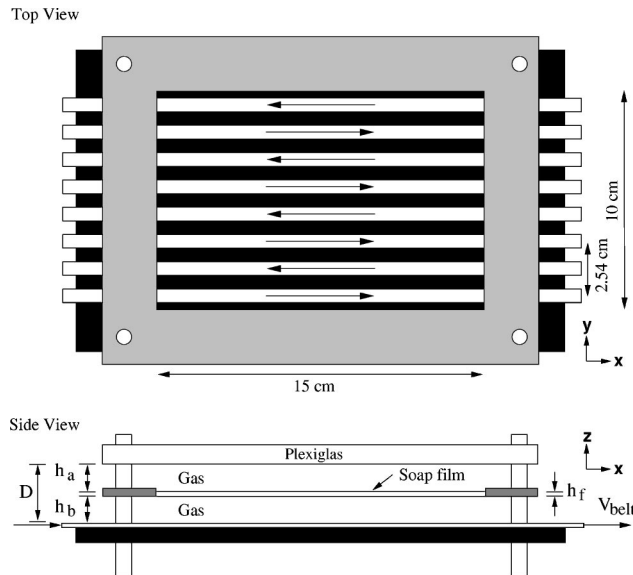


FIG. 2. Top view: stainless steel belts (0.01 cm thick and 0.63 cm wide) move at a constant, adjustable speed, but alternate in direction. Side view: gas surrounds the film both above and below. The Plexiglas plate suppresses the growth of the boundary layer in the gas above the film, and an aluminum plate below the belts and the belts themselves impose a similar condition below the soap film. The distance between the aluminum and Plexiglas plates is $D = h_a + h_f + h_b$. The film thickness h_f and the belt thickness are not drawn to scale.

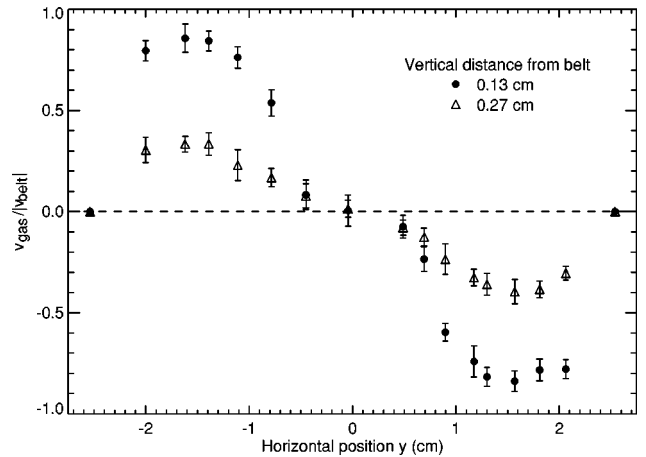


FIG. 3. Experimental measurements with a hot-film anemometer of the horizontal velocity profile in the gas without a soap film present. The forcing very near the belts is periodic but not sinusoidal. Above the belts, the high frequency modes in the shape of the horizontal velocity profile are viscously damped, leaving a profile more closely approximating a sinusoid.

in contact with an aluminum plate to reduce vibration. The pulleys are driven by a stepper motor.

We suspend a Plexiglas plate a distance h_a (adjustable in the range $0.2\text{--}1.0 \text{ cm}$) above the film to suppress the growth of the Blasius boundary layer in the gas, thus imposing a two-dimensional Couette velocity profile in the gas above the film. Hot film anemometry measurements of the vertical velocity profile in the gas between the belts and the Plexiglas plate in the absence of the soap film confirm the linearity of the profile (Fig. 4). The importance of this well-defined linear velocity profile is discussed in Sec. IV.

The film is created by submerging a rectangular metal frame with a $10 \text{ cm} \times 15 \text{ cm}$ hole in a reservoir of 1% by volume commercial liquid detergent (Ivory brand, manufactured by Proctor and Gamble), 10% glycerol, and 89% distilled water. This concentration of surfactant corresponds to 30% of the critical micelle concentration. Upon removal

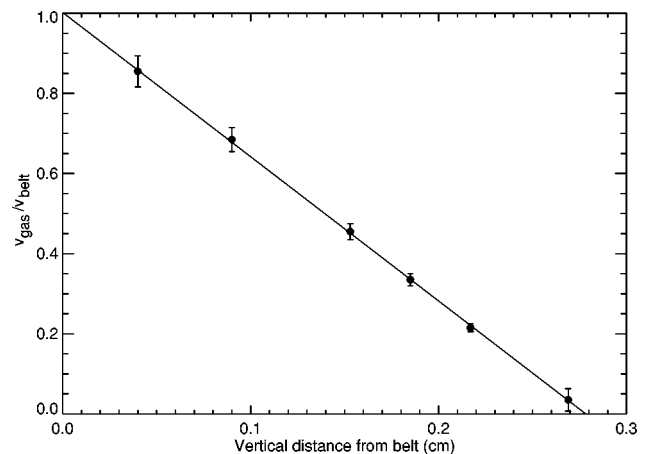


FIG. 4. Experimental measurements with a hot-film anemometer of the z dependence of the x component of velocity without a soap film present. A rigid Plexiglas plate (0.28 cm above the belts) suppresses the growth of the boundary layer and results in a linear velocity profile in the gas.

TABLE I. Dynamic and kinematic viscosities for several gases saturated with water vapor, as measured with a capillary tube viscometer at 1 atm and 24 °C.

Gas	μ_g (10^{-5} g cm $^{-1}$ s $^{-1}$)	ν_g (cm 2 s $^{-1}$)
Helium	20.0	1.09
Argon	22.2	0.12
Nitrogen	17.7	0.15
Carbon dioxide	15.0	0.084

from the reservoir, the frame has a flat soap film stretched across the hole. When oriented horizontally, as in the experiment, the film sags approximately 0.05 cm under its own weight.

Visualization of the flow is achieved with illumination by a monochromatic sodium lamp. Random thickness fluctuations formed at the creation of the film provide reflected intensity variations via the interference pattern created by the optical path length difference between the two monolayers of surfactant (see Fig. 1). The flow is observed by following fluid elements of constant thickness, which are advected by the flow due to the separation of time scales mentioned above.

The apparatus is encased in a sealed glass and Plexiglas box to suppress drafts and dust contamination and to maintain a high-humidity environment. Slowing the rate of evaporation is an important experimental consideration, as the two-dimensional density of the film and the film viscosity depend on the thickness of the film. Evaporation can be slowed only by maintaining a high-humidity environment, because the soap film is freely suspended and not in contact with a reservoir from which it could replace lost mass. In a high-humidity environment, films can be studied for hours with no

appreciable change in properties.

The sealed box also allows us to study the effect of changing the gas contained within. We use helium, nitrogen, argon, and carbon dioxide for their range of viscosities (Table I). The viscosity of the gas is measured with a capillary tube viscometer, in which the decay of an applied pressure gradient is measured with a piezoelectric pressure sensor.

The effects of variation in the viscosity of the gas surrounding a soap film on the flow of the film have not been previously examined. However, a gas pressure dependence on soap film flows has been reported by Rutgers *et al.* [4], as discussed in Sec. IV.

A thermistor thermometer is used to monitor the temperature inside the box, as the system is continually heated by two sodium lamps used for illumination and cooled by the injection of compressed gases. Experiments were performed at a temperature of (24 ± 2) °C. The roof of the box is constructed from two glass windows which allow the sodium light to illuminate the film and a charge-coupled diode (CCD) camera to record the film behavior from above. Images of the film are recorded onto a personal computer with the CCD camera at a rate of 15 frames per second, with each frame containing 512×256 , 8-bit pixels.

III. EXPERIMENTAL RESULTS

By changing the gas surrounding the soap film and the thickness of the gas layers (h_b and h_a), the onset of instability of the soap film flow changes in a manner demonstrated in Fig. 5. We can see a general trend for the dependence of onset belt velocity on h_b and h_a : the belt velocity must be increased to destabilize the flow as either h_a or h_b is decreased. In both cases, the Plexiglas plate is moving closer to

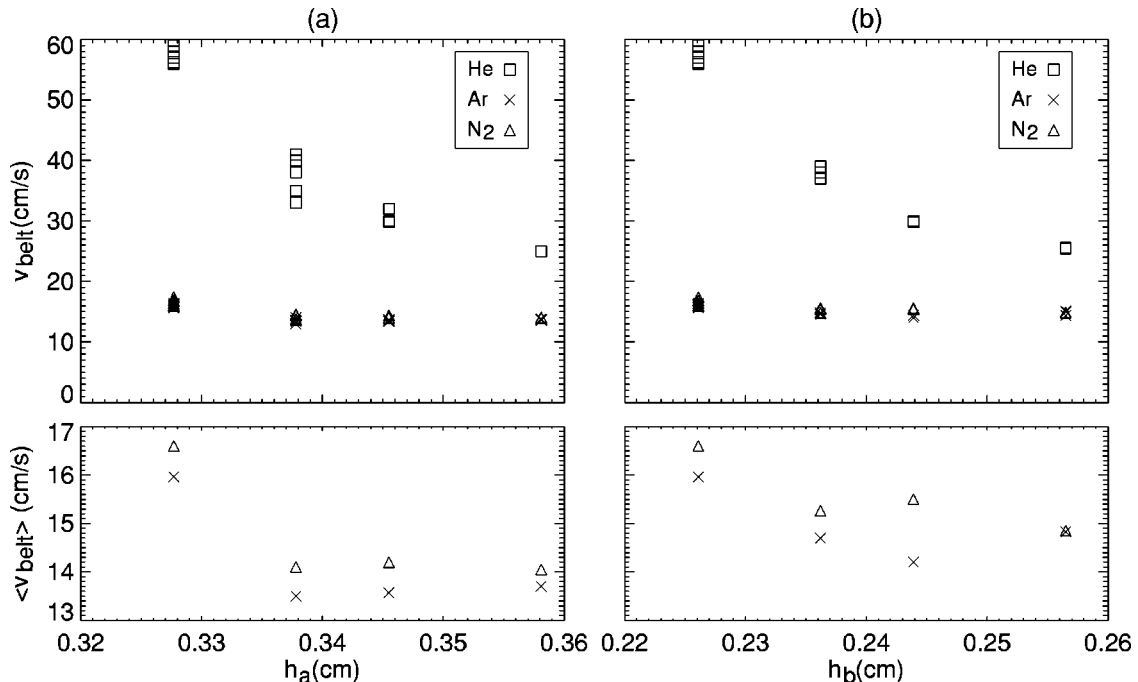


FIG. 5. (a) Belt velocity at the onset of instability as a function of the thickness h_a of the upper gas layer for argon, helium, and nitrogen gases, where $h_b = 0.23$ cm. (b) Belt velocity at the onset of instability as a function of the thickness h_b of the lower gas layer for the same gases, where $h_a = 0.33$ cm. The lower plots show the belt speed at onset for argon and nitrogen.

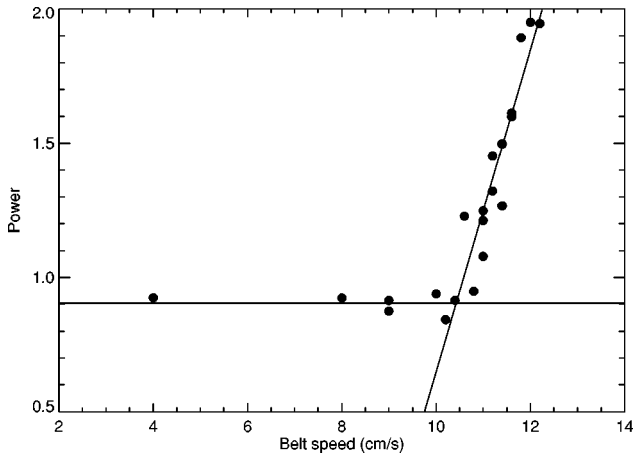


FIG. 6. Determination of onset from the sum of the amplitudes squared of modes 3 and 4 in the power spectra. The belt speed at onset for this set of parameters (nitrogen, $h_b=0.40$ cm, $h_a=0.52$ cm) is 10.4 ± 0.5 cm/s.

the belts (D is decreasing), which increases the drag on the film.

We determine the onset of the flow instability by taking the two-dimensional fast Fourier transform (2DFFT) of single images at different belt speeds with all other parameters held constant. Rows of three or four vortices in the images correspond to low-frequency modes in the 2DFFT. The power in modes 3 and 4 is summed to give the amplitude squared of the pattern.

We plot the power in modes 3 and 4 as a function of belt speed in Fig. 6. We fit the power versus belt speed well below onset to a horizontal line. We fit a second line to the data well above onset. The point at which these two lines cross is taken to be the belt speed at which onset of the primary instability has occurred. In Fig. 6, we see that the data follow a straight line above onset. The linear growth of squared amplitude above onset and a lack of observed hysteresis indicate that the system undergoes a supercritical bifurcation. The onset belt speeds obtained by this method correspond closely to speeds obtained through a visual determination of the onset of fluid motion transverse to the direction of forcing and are reproducible to within 5%.

The simplest attempt to describe this system, i.e., modeling the soap film as a strictly two-dimensional flow decoupled from the surrounding gas, does not involve parameters such as the thickness of the gas layers or the viscosity of the gas. With a Reynolds number defined in such a manner, $R_{\text{expt}} = v_{\text{film}}/k\nu_f$, where k is the forcing wave number and ν_f is the kinematic viscosity of the film, we can calculate a critical value to compare to linear stability analysis. Assuming a linear variation of velocity from the belts to the Plexiglas plate with the soap film present, and experimentally setting $h_a=h_b$, we let $v_{\text{film}}=v_{\text{belt}}/2$. For $v_{\text{belt}}=10$ cm/s (in nitrogen, for example), $k=2.47$ cm $^{-1}$, $\nu_f=0.03$ cm 2 /s, and $R_{\text{expt}} \approx 70$. (This ν_f was reported by Couder and Basdevant [1].) Linear stability analysis for this strictly two-dimensional system predicts a critical Reynolds number $R_c = \sqrt{2}$. This factor of 50 discrepancy cannot simply be ascribed to an incorrect value of ν_f . Since the measured belt speeds at onset of instability change with gas and geometric parameters as shown in Fig. 5, the dimensionless parameters

in our system must depend on both the gas and the thickness of the gas layers.

Bondarenko *et al.* [13] encountered a similar situation in a magnetohydrodynamic realization of the Kolmogorov flow; the friction between the fluid and the supporting glass plate played a significant role in the stability of the quasi-two-dimensional flow. They found that by including consideration of a frictional force acting from the third dimension in the equations of motion, they could extend the theoretical model to account for the discrepancies between the experimental ($R_c \approx 10^3$) and previous theoretical results ($R_c = \sqrt{2}$). In our experimental system, although the flow field in the film is two dimensional, we clearly see a variation in onset belt speed when the parameters corresponding to influences of the gas in the third dimension are varied.

IV. MODEL

To understand the stability of the Kolmogorov flow in the soap film, we need a model that includes the three-dimensional drag effects of the gas on the film. Couder *et al.* [8] proposed that the force on the soap film due to the surrounding gas may be treated as a frictional force given by $\mu_g dU/dz$, where μ_g is the dynamic viscosity of the gas and dU/dz is the derivative of the gas velocity tangential to the film. In our experiment, the Plexiglas plate parallel and close to the soap film leads to a linear velocity variation with z , as Fig. 4 demonstrates, so that the expected frictional force is $\mu_g v_{\text{film}}/h_a$. This conflicts with our data (Fig. 5), which suggest that the belt speed at onset is related to the kinematic rather than dynamic viscosity of the gas. Table I shows that argon has a higher dynamic viscosity than helium, but a lower kinematic viscosity. Our observation that the belt speeds at onset of instability in argon for different h_a , h_b are consistently lower than those in helium indicates that the kinematic viscosity and hence the density of the gas surrounding the soap film should be considered.

This is further supported by experiments varying gas pressure by Rutgers *et al.* [4]. When the pressure near the soap film was lowered to the vapor pressure of water (≈ 25 Torr), the drag on the film became small. Since dynamic viscosity does not change with pressure until less than 4 Torr, the change in gas density must be responsible for the change in friction between the film and the gas. This also implies the kinematic viscosity is the relevant parameter for this frictional effect.

In light of these observations, we propose the following model. Rather than treating only the film as a two-dimensional fluid interacting with the third dimension, we treat the entire region between the moving belts and the Plexiglas plate as a two-dimensional fluid. Starting from the three-dimensional Navier-Stokes equations, we assume velocity, density, and viscosity profiles in the z direction and integrate over z to produce two-dimensional equations. This procedure is similar to that used to calculate two-dimensional equations of motion for the flow of fluid confined between two parallel plates [16]; although the velocity field may be considered two-dimensional, friction with the bounding walls plays an important role in the flows [15,17].

The flow of gas and film may each be described by the three-dimensional Navier-Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla P}{\rho} + \nu \nabla^2 \mathbf{u}, \quad (1)$$

where $\mathbf{u}(x, y, z)$ is the three-dimensional fluid velocity, P is the pressure, and $\rho(z)$ and $\nu(z)$ are the fluid densities and kinematic viscosities and depend only on z . We assume that the velocity \mathbf{u} may be treated as the product of a two-dimensional velocity field $\mathbf{v}(x, y) \perp \hat{z}$ and a profile $f(z)$, where the film lies in the x - y plane. Because the soap film is highly resistant to shear in the z direction, we take the velocity to be independent of z within the film; in the gas regions, we allow the velocity to vary linearly with z , as measurements without the soap film indicate (Fig. 4). This defines our profile as

$$f(z) = \begin{cases} [1 - z(1 - \gamma h_a/D)/h_b], & 0 < z < h_b, \\ \gamma h_a/D, & h_b < z < h_b + h_f, \\ \gamma[1 - z/D], & h_b + h_f < z < D, \end{cases} \quad (2)$$

where $z=0$ corresponds to the location of the belts, $D \equiv h_a + h_b + h_f$, $h_f \ll h_a, h_b$, and $\gamma h_a/D = v_{\text{film}}/v_{\text{belt}}$. The slopes of the velocity profiles in the gas layers on either side of the soap film have different values due to the drag within the film. If the slopes were identical, the velocity profile across the three-layer system would be linear rather than piecewise linear and γ would be unity. The value of γ differs from this value in our experiment and is determined from direct observation of the film speed. This definition of $f(z)$, which corresponds to keeping only the first-order terms in expansions in z for each layer, does not conserve the tangential stress across the soap film, and is therefore unphysical. However, using a piecewise-linear approximation to the actual (unknown) velocity profile allows us to simplify the three-dimensional, three-fluid model to a single two-dimensional equation of motion.

To arrive at a two-dimensional equation, we substitute $\mathbf{u}(x, y, z) = f(z)\mathbf{v}(x, y)$ into Eq. (1) and average over z ; i.e., we take $(1/D)\int_0^D dz$ of the resulting equation. Most of this calculation is straightforward, with the only subtlety occurring in the term $\nu(z)\nabla^2 f(z)$. At the gas-film boundaries $f(z)$ is continuous, but the effectively infinite resistance to shear between the two surfactant layers makes $df(z)/dz$ discontinuous, so that d^2f/dz^2 is a Dirac delta function at these boundaries. Integration over z then requires us to evaluate ν at the gas-film boundary. Since we expect that the greater susceptibility to shear of the gas will cause most of the velocity variation to occur there, rather than in the film, we assign the viscosity of the gas to the viscosity at the boundary. This then produces the two-dimensional equation of motion for columns of gas and film,

$$\frac{\partial \mathbf{v}}{\partial t} + N(\mathbf{v} \cdot \nabla_{\perp})\mathbf{v} + \int_0^D \frac{\nabla_{\perp} P}{D\rho} dz = A\nabla_{\perp}^2 \mathbf{v} - B\mathbf{v}, \quad (3)$$

$$N \equiv \frac{2}{3} \left[\frac{h_b}{(h_b + \gamma h_a)} + \frac{\gamma h_a}{D} \right], \quad (4)$$

$$A \equiv \nu_g + \frac{2\nu_f h_f \gamma h_a}{D(h_b + \gamma h_a)}, \quad (5)$$

$$B \equiv \frac{2\nu_g(1 - \gamma)}{h_b(h_b + \gamma h_a)}. \quad (6)$$

Integration of the three-dimensional equations over the z direction has produced an extra term in the two-dimensional equation of motion, $-B\mathbf{v}$; this term, proportional to the two-dimensional velocity, plays the role of an extra drag force. Such a term was also derived in the model of Kolmogorov flow in a thin layer of electrolytic fluid, where it was responsible for an increase in the critical fluid velocity, as mentioned in the previous section [13].

We incorporate the forcing by adding a body force to the two-dimensional equation of motion, rather than by enforcing a boundary condition on the three-dimensional system prior to integration. This is accomplished through the addition of a term $F_0 \sin(ky)\hat{\mathbf{x}}$ to Eq. (3). Given this forcing, a steady-state solution $v_{\text{belt}} \sin(ky)\hat{\mathbf{x}}$ exists when $F_0 = (B + Ak^2)v_{\text{belt}}$.

We nondimensionalize Eq. (3) plus the additional forcing term with the wave vector k and a velocity scale v_{belt} . This leads to the two-dimensional equation of motion

$$\begin{aligned} \frac{\partial \mathbf{v}'}{\partial t'} + N(\mathbf{v}' \cdot \nabla'_{\perp})\mathbf{v}' + \int_0^D \frac{\nabla'_{\perp} P}{v_{\text{belt}}^2 k \rho} dz \\ = -\frac{1}{Q}\mathbf{v}' + \frac{1}{R}\nabla'_{\perp}{}^2 \mathbf{v}' + \left(\frac{1}{R} + \frac{1}{Q} \right) \sin y' \hat{\mathbf{x}}, \end{aligned} \quad (7)$$

where primed variables are nondimensional. Thus there are two dimensionless control parameters

$$R = v_{\text{belt}}/Ak = \frac{v_{\text{belt}}}{k[\nu_g + 2\nu_f h_f \gamma h_a/D(h_b + h_a \gamma)]}, \quad (8)$$

$$Q = v_{\text{belt}}k/B = \frac{v_{\text{belt}}kh_b(h_b + h_a \gamma)}{2\nu_g(1 - \gamma)}. \quad (9)$$

V. DISCUSSION

Our choice of velocity scale has allowed us to cast the problem so that the coefficient of the forcing term $\sin y'\hat{\mathbf{x}}$ is the sum of the coefficients of the linear friction term $-\mathbf{v}'$ and the Reynolds friction term $\nabla'_{\perp}{}^2 \mathbf{v}'$. In this form, the problem of the stability of Kolmogorov flow in a plane with a linear drag force has been studied by Thess [14], and we may compare our experimental results to his calculations. Note that by using a more complicated velocity scale for nondimensionalization, we could set $N=1$, corresponding directly to the numerical results presented by Thess. Such a choice results in a different scale factor for R and Q , but does not alter the physical results. For clarity, we have chosen a simpler representation for this discussion.

For each experiment, ν_g , h_a , and h_b (and therefore D) are fixed. We measure γ by finding the film velocity over the center of a belt. For one set of h_a and h_b , in nitrogen, $\gamma = 0.83 \pm 0.02$ for several different belt speeds. For simplicity, we use this value in our calculations as a constant for all experiments. A more sophisticated model could include γ as a function of h_a , h_b , and ν_g , although such considerations may be overshadowed by the inaccuracy of the piecewise-linear approximation of the vertical velocity profile.

The only undetermined parameter in our model is $h_f \nu_f$, the product of the film thickness and film viscosity [Eq. (8)]. Because both R and Q depend linearly on v_{belt} , their ratio is independent of v_{belt} . An onset observation involves fixing all experimental parameters but belt velocity and then ramping belt velocity to move radially outward from the origin in the R - Q plane along the line with slope

$$R/Q = \frac{2(1-\gamma)}{k^2 h_b (h_b + h_a \gamma)} \frac{1}{1 + 2(\nu_f/\nu_g) h_f h_a \gamma / D (h_b + h_a \gamma)} \quad (10)$$

until the flow becomes unstable to a pattern. The process is then repeated for a variety of gases and geometric parameters. By fitting the experimental data in nitrogen to the linear stability results, we determine $h_f \nu_f \approx 0.63 \text{ cm}^3/\text{s}$. So if h_f is typically $3 \text{ }\mu\text{m}$, $\nu_f \approx 2100 \text{ cm}^2/\text{s}$. As mentioned above, Couder and Basdevant [1] reported a film viscosity of $\nu_f = 0.03 \text{ cm}^2/\text{s}$ for a $10 \text{ }\mu\text{m}$ soap film (assuming the film density is equal to that of water). Beizaie and Gharib [5] reported values from 0.0375 to $0.1072 \text{ cm}^2/\text{s}$ for film thicknesses from 5.4 to $1.6 \text{ }\mu\text{m}$ respectively. Martin and Wu [18] reported film viscosity values at least 10^2 times larger than those of Refs. [1] and [5] for a 0.5 – $1.0 \text{ }\mu\text{m}$ film. This large discrepancy between reported values of soap film viscosity and our fit value suggests that our model overestimates the drag forces of the gas on the film. For the model to properly account for the effects of the film viscosity, its value has been inflated by the fitting procedure.

Figure 7 shows the comparison between experiment and theory for several values of h_a , h_b , and ν_g . In the high- Q limit that would correspond to models of Kolmogorov flow which lack a friction term linearly proportional to the velocity, the stability boundary asymptotically approaches a value $R_c \approx 4.1$. This asymptotic value corresponds to $R_c = \sqrt{2}$ in the simplest model, but has a different value since the nonlinear term in our model has a coefficient $\approx 2/3$ rather than unity. In the low- Q limit, where gas kinematic viscosity dissipation dominates the flow of the three-fluid-layer system, R_c becomes unbounded. In this limit, the systematic deviation of our data from the linear stability curve becomes large.

The large value of the soap film viscosity in the model and its systematic deviation from the data in the high-gas-viscosity limit show inadequacies in the model. We have made several assumptions and approximations in the process of modeling this soap film. We have assumed that γ is independent of h_a , h_b , and ν_g . Our lack of direct film velocity data makes measurement of this parameter difficult. We have observed, however, that γ does not depend on the velocity of the belts. We have measured that the velocity profile in the gas between two rigid plates is linear. However, as the pres-

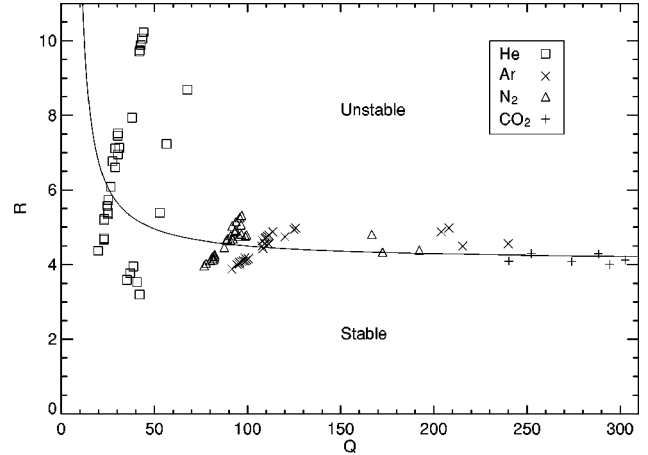


FIG. 7. Comparison of onset of instability from experiment and theory for different gases and different h_b and h_a . $1/R$ is the coefficient of the Reynolds friction term in the equation of motion, and $1/Q$ is the coefficient of the friction due to the viscous coupling between the gas and film. Above the curve, the flow is unstable; below the curve, the flow is stable. Each experimental point corresponds to the R, Q value at which the flow in a particular experiment became unstable.

ence of the hot-film anemometer destroys the soap film, we cannot measure the velocity profile while the film is in place. We assume, then, that the velocity profile in the gas is linear. Also, the horizontal velocity profile in the gas is demonstrated to not be a pure sinusoid, as we assume in the model. This may have an effect on the comparison between the experiment and the model.

Finally, and most likely, there may be a three-dimensional flow associated with onset that cannot be captured by quasi-two-dimensional modeling. We now briefly discuss a succession of increasingly sophisticated two-dimensional models to describe the three-dimensional effects evident in the soap film system. The simplest model, as discussed in Sec. III, completely decoupled the flow of the film from the flow of the surrounding gas. This analysis led to a disagreement between theory and experiment by a factor of 50.

A next step, not previously discussed in this paper, is to apply the approach of Couder *et al.* [8], in which the film is coupled to the gas, but the gas plays a purely passive role. Assuming that the velocity profiles in the gas are linear then leads to equations of motion for the film only; these equations include a drag term linear in the velocity. Stability analysis of these equations fails to even qualitatively reproduce the experimental data.

These considerations led us to develop the model outlined in Sec. IV, in which two-dimensional equations of motion for gas-film-gas columns are derived and studied. While the agreement is better than for previous models, it has the inadequacies described above, suggesting that the role of the gas is still not properly represented.

Probably only a full three-dimensional analysis of the three-fluid system can completely describe the experiment. While such an analysis would be extremely laborious, it would be quite useful: satisfactory agreement with experiment would provide the first measurement of soap film viscosity that properly accounts for the effects of the film-gas

coupling. Further, preliminary calculations along these lines strongly suggest that the flow does not remain quasi-two-dimensional. Even if the perturbations are initially quasi-two-dimensional, flow incompressibility appears to drive flows in the third dimension.

Although we cannot completely account for the effects of the viscous coupling of the gas to the soap film, our results clearly show that they cannot be neglected. We conclude that comparisons of soap films to idealized two-dimensional

flows must be conducted with care to avoid misinterpretation.

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